

# Test Paper of Wuhan University of Technology (A)

2018-2019 Academic Year 1<sup>st</sup> Semester, Course: \_\_\_\_\_,

Closed-book Exam

Time: 120mins, Class hours: \_\_\_\_\_ Credits: \_\_\_\_\_ Total Points: 100, Percent in final evaluation

result: \_\_\_\_\_ % Date: \_\_\_\_\_

| No.        | 一<br>I | 二<br>II | 三<br>III | 四<br>IV | 五<br>V | 六<br>VI | 七<br>VII | 八<br>VIII | 九<br>IX | 十<br>X | 合计<br>Total | Teacher |
|------------|--------|---------|----------|---------|--------|---------|----------|-----------|---------|--------|-------------|---------|
| Full score |        |         |          |         |        |         |          |           |         |        | 100         |         |
| Score      | 10     | 15      | 10       | 29      | 12     |         |          |           |         |        | 75          |         |

得分  
Score

I. Single-choice questions (2 points \* 5=10 points)

1. (A) The basic characteristics of elastic elements are

- $k = \frac{F}{\Delta x}$
- A. the elements that represent the relationship between force and displacement
  - B. the elements that represent the relationship between force and speed
  - C. the elements that represent the relationship between force and acceleration
  - D. the elements that represent the relationship between displacement and speed

2. (A) For the forced vibration under the simple harmonic excitation of the single-degree-of-freedom system, the magnitude of the response amplitude in the low-frequency zone, resonance zone and high-frequency zone mainly depends the system's

- A. spring stiffness, damping and inertia
- B. damping, spring stiffness and inertia
- C. inertia, spring stiffness and damping
- D. inertia, damping and spring stiffness

3. (B) The function that best reflects the correlation of random variables at different times in different random processes is

- A. autocorrelation function
- B. cross-correlation function
- C. auto-spectrum
- D. frequency response function

4. (C) There are mainly three kinds of vibration problems. Besides vibration analysis and prediction of vibration environment, what is the other vibration problem?

- A. vibration prediction
- B. vibration modeling
- C. system identification
- D. modal analysis

5. (A) The function that cannot reflect the inherent characteristics of the system is

- A. autocorrelation function
- B. impulse response function
- C. frequency response function
- D. natural frequency

得分  
Score

II. Explanation of phases (5 points \* 5=25 points)

6. Sample function

由多个样本共同组成的函数, 包含所有情况下样本的可能性, 能直观反映大体的样本变化。

随机函数构成, 每一个样本函数都随机产生。

7. Multi-degree-of-freedom vibration system

由多个自由度组成的振动系统, 能反应出振动系统在空间上的振动变化。

由多个独立坐标确定的位置系统

# 8. Power spectrum

功率谱函数与系统特征可推知输入功率谱

在频率上表现随机振动过程的特征。

# 9. Spectral analysis

振动系统中的物理量与时间相关的函数。

分析不同频率谐波的振幅化。

# 10. Principle of dynamic vibration absorber

利用动力振动来与一部分振动相抵消的原理。

对用辅助质量的动力作用,使弹性动作在主振件上。  
的力与主振件振幅正好大小相等与主振件方向相反。

III. Short-answer questions (10 points \* 2 = 20 points)

得分  
Score 10

# 11. Give examples to illustrate the similarities and differences between active vibration isolation and passive vibration isolation.

相同点: 都是由单自由度变为二自由度的分析。

不同点: 主动隔振是在初始受力后, 不受外界影响的隔振。

被动隔振受外界激励的影响。

# 12. What are the concepts of impulse response function and frequency response function and their relationship?

脉冲响应函数: 系统振动时频响函数在坐标轴上呈  $\sin x$  或  $\cos x$  形式变化的函数。  
频率响应函数: 系统振动时描述函数频率的函数。

关系: 脉冲响应函数与频率响应函数互为傅氏变换。  $Y(\omega) = H(\omega) X(\omega) \Rightarrow H(\omega) = \frac{Y(\omega)}{X(\omega)}$

共同点: 描述振动系统的振动特性。

不同点: 前者在频域上描述系统动态特征, 后者在时域上描述系统动态特征。

得分  
Score 28

# IV. Calculation (10 points \* 3 questions = 30 points)

# 13. The probability density function of random variable X is $p(x) = ae^{-b|x|}$ , of which a and b are constants. Find the relationship between a and b, and when a = 1, determine the distribution function F(x).

解:  $\phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{t^2}{2}} dt$

$$\therefore F(x) = P\{x \leq X\} = P\{-\infty \leq X \leq x\} = \int_{-\infty}^x p(x) dx.$$

$$\therefore F(x) = \int_{-\infty}^{0+} ae^{-bx} dx + \int_0^{\infty} ae^{bx} dx$$

$$= -abe^{-bx} \Big|_{-\infty}^{0+} + bae^{bx} \Big|_0^{\infty} = 1$$

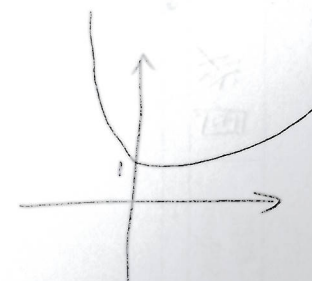
$$\Rightarrow ab = 1$$

$$\Rightarrow \text{当 } a=1 \text{ 时, } b=1 \quad p(x) = e^{-|x|}$$

$$F(x) = \int_{-\infty}^{0+} e^{-x} dx + \int_0^{\infty} e^x dx$$

$$= -e^x \Big|_{-\infty}^{0+} + e^x \Big|_0^{\infty}$$

$$=$$



14. It is already known that  $R(x) = 25e^{-4|x|} \cos \omega \tau + 16$ , work out  $\mu(x)$ ,  $\sigma_x^2$ ,  $S_x(\omega)$ .

解:  $\mu_x^2 = \lim_{v \rightarrow \infty} R_x(v) = 16$   $E[x^2] = \lim_{v \rightarrow \infty} R(v) = 41$   
 $\sigma_x^2 = E[x^2] - \mu_x^2$   $S_x(\omega) = F[R_x(v)] = F[25e^{-4|v|} \cos \omega v + 16]$   
 $= F[25e^{-4|v|} \cos \omega v] + F[16]$   
 $= \frac{25}{4} \cdot \frac{\omega^2 + 8}{\omega^2 - 4} + 32\pi \delta(\omega)$

15. Take the linear displacement of  $m_2$  as the coordinates of motion to determine the equivalent stiffness  $k_B$  and equivalent mass  $m_B$  of the system and natural frequency of vibration. Each value in the figure is known, regardless of the mass of the horizontal lever and the distributed mass of the elastic element.

Solution:

Suppose the movement of  $m_2$  is  $x$ , thus 用能量法求  $k_B$

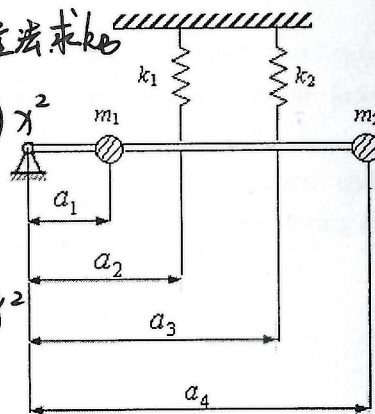
$$U_a = \frac{1}{2} k_1 \left( \frac{a_2}{a_4} x \right)^2 + \frac{1}{2} k_2 \left( \frac{a_3}{a_4} x \right)^2 = \frac{1}{2} \left( \frac{a_2^2 k_1 + a_3^2 k_2}{a_4^2} \right) x^2$$

since  $U_e = \frac{1}{2} k_B x^2$ , thus  $k_B = \frac{a_2^2 k_1 + a_3^2 k_2}{a_4^2}$

$$T_a = \frac{1}{2} m_1 \left( \frac{a_1}{a_4} \dot{x} \right)^2 + \frac{1}{2} m_2 \dot{x}^2 = \frac{1}{2} \left( \frac{a_1^2}{a_4^2} m_1 + m_2 \right) \dot{x}^2$$

since  $T_e = \frac{1}{2} m_B \dot{x}^2$ , thus  $m_B = \frac{a_1^2}{a_4^2} m_1 + m_2$

$$f_B = \sqrt{\frac{k_B}{m_B}} = \sqrt{\frac{a_2^2 k_1 + a_3^2 k_2}{a_1^2 m_1 + a_4^2 m_2}}$$



得分  
Score

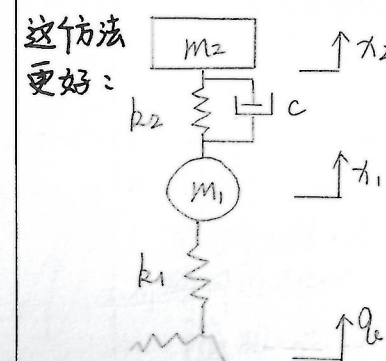
12

V. Comprehensive problem-solving questions (15 points)

16. There are two methods to simplify the car into a two-degree-of-freedom vibration system model in the course. Draw the diagrams of two-degree-of-freedom vibration system models under the two methods; write down their respective assumptions for simplification and explain the physical meaning of each value in the diagrams. If you want to analyze the deformation and dynamic load of suspension and tire under irregular road (displacement) excitation, which two-degree-of-freedom model is best used? Deduce the corresponding vibration differential equation of this model.

Solution:

这个方法更好:



$m_2$  — 悬挂质量

$m_1$  — 非悬挂质量

$k_2$  — 悬挂刚度

$k_1$  — 轮胎刚度

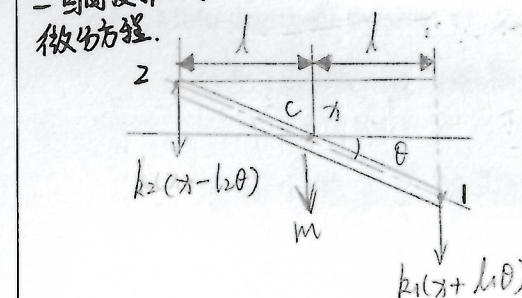
$c$  — 减振器阻尼系数

$q$  — 路面不平激励

$$\begin{cases} m_2 \ddot{x}_2 + c(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = 0 \\ m_1 \ddot{x}_1 + c(\dot{x}_2 - \dot{x}_1) + k_2(x_1 - x_2) + k_1(x_1 - q) = 0 \end{cases}$$

$$\begin{cases} q(t) = e^{i\omega t} \\ x_1(t) = H(\omega) x_1 \sim q e^{i\omega t} \\ \dot{x}_1(t) = i\omega H(\omega) x_1 \sim q e^{i\omega t} \\ \ddot{x}_1(t) = -\omega^2 H(\omega) x_1 \sim q e^{i\omega t} \\ x_2(t) = H(\omega) x_2 \sim q e^{i\omega t} \\ \dot{x}_2(t) = i\omega H(\omega) x_2 \sim q e^{i\omega t} \\ \ddot{x}_2(t) = -\omega^2 H(\omega) x_2 \sim q e^{i\omega t} \end{cases}$$

= 自由度振动微分方程



$k_2(x - l_2\theta)$  — 2点的受力

$k_1(x + l_1\theta)$  — 1点的受力

$$\begin{cases} m \ddot{x} = -k_1(x + l_1\theta) - k_2(x - l_2\theta) \\ J \ddot{\theta} = k_1 l_1 (x + l_1\theta) + k_2 (x - l_2\theta) \end{cases}$$